Computational Thinking in Mathematics Education: A Joint Approach to Encourage Problem-Solving Ability

Erick John Fidelis Costa, Livia Maria Rodrigues Sampaio Campos and Dalton Dario Serey Guerrero
Software Practices Laboratory (SPLab) and Distributed Systems Lab (LSD)
Federal University of Campina Grande (UFCG), Paraiba, Brazil
erick.costa.ti@gmail.com, livia@computacao.ufcg.edu.br, dalton@computacao.ufcg.edu.br

***Abstract*—In this paper, we analyzed whether the ability of students to solve problems can be influenced by the implementation of Computational Thinking (CT) in the teaching of mathematics. In this sense, we performed a quasi-experiment with students in the Brazilian's Basic Education Regular System (particularly in a group of students around fifteen years old) comparing their performance after being trained with practical activities using exercises in more or less conformity with well known CT concepts and capabilities. Considering the quantitative results obtained, the experimental group outperformed the control group, which worked with exercises in more and less conformity with CT, respectively. The results were statistically significant, suggesting that bringing together CT and mathematics through proper adjustments of classroom practices can have a positive influence on students abilities of solving problems. In order to strengthen this conclusion we also performed a qualitative analysis comprising students and the teacher involved in the quasi-experiment that reinforced our quantitative results.**

***Keywords*—computational thinking; teaching of mathematics; problem solving ability**

I. Introduction

According to Wing [1], Computational Thinking (CT) can be described as an analytical approach that shares the methods and procedures of mathematical thinking, as well as other ways people usually apply in problem solving, such as logical thinking. This approach can influence several research fields and is intended for everyone, everywhere, because it is considered an intrinsic approach and general character that can be applied outside the computer science limits. The main characteristic abilities of CT are:

- Formulating problems in such a way that enables people to use a computer and other tools to help on solving them;
- Logically organizing and analyzing data;
- Representing data through abstractions such as models and simulations;
- Automating solutions through algorithmic thinking (a series of ordered steps);
- Identifying, analyzing, and implementing possible solutions with the goal of achieving the most efficient and effective combination of steps and resources;
- Generalizing and transferring this problem solving process to a wide variety of problems.

The basis of CT comes from how professionals from computer and related areas understand and plan solutions to problems, mapping and connecting layers of the proposed context to deploy viable solutions [2]. In the literature of CT two approaches are highlighted: teaching disciplines or courses in computing (e.g. programming, robotics and games development) with or without the use of computers; and the introduction of computer science concepts combined with the disciplines of the basic education cycle.

The first approach aims to develop CT using mechanisms such as programming (algorithms and procedures) [3] and unplugged computing. This second option concerns the practice of making people understand concepts of computer science (e.g. network protocols and binary numbers) with playful elements like games and puzzles without using a computer, that is an unplugged solution [4], [5].

Following the first approach requires the introduction of a specific discipline for learning/teaching computing concepts in the curriculum of the school, what may not be possible. In the public Brazilian educational system, for example, one reason is the already overloaded curriculum [6].

On the other hand, the application of concepts of computer science to the disciplines of the basic education cycle takes place by combining these concepts with the activities that are part of the methodology designed to each discipline in any educational environment [7], [8]. This may favor the development of CT in basic education since it does not require an specific computing discipline or professionals. However, the main challenge in this sense is how to implement such an approach.

In order to support the development of CT in basic education without the need of a specific computing discipline, Barr and Stephenson [9] suggested some skills considered the core of CT. These skills should be stimulated in the context of other the disciplines of basic education, such as mathematics, science and reading. Here are the skills suggested by the authors: data collection, data analysis, data representation, decomposition, abstraction, algorithms, automation, parallelization and simulation.

The objective of the work presented in this paper was to assess and demonstrate that CT skills [9] can be developed in

978-1-5090-5920-1/17/$31.00 ©2017 IEEE

mathematics students without the need of specific disciplines in computing but by means of exercises of questions worked in classroom that are more aligned to CT and problem solving. In this sense, we conducted a quantitative analysis to compare the performance of students on solving problems after participating in a training with exercises of questions more and less aligned to CT.

We also conducted a qualitative analysis to figure out the vision of the students and teacher who were involved in the study. This work resembles the one presented by Lima [10] when he proposed the combination of important pedagogical strategies to improve problem solving student abilities, on the other hand our study was conducted in a more detailed way from a more practical point of view.

The quantitative analysis was based on a quasi-experiment [11], [12] with Brazilian students in the 8th grade (under fifteen years old) which were divided into control and experimental groups; we chosen a quasi-experiment because we preferred to randomly assign school classes to be control and experimental, but we did not choose the participants that belonged to each group. This decision was taken priorly by the school enrollment process. Despite that, students metrics were analyzed and we claim the distribution is fairly random. The students in the control group participated of a training with traditional math questions prepared by their teachers and used in their daily practice, while the experimental group worked with the the same professor, same methodology but using questions that were adapted or prepared in order to be better aligned with CT skills.

In order to defined how aligned a math question would be with CT skills we used the approach proposed by Mestre et al. [13]. The authors define that the more CT skills covered in a question the more aligned it is with CT according to a mapping from CT skills to fundamental mathematical skills based on [9].

We compared the performance of the control and experimental groups of students in terms of the problem solving abilities of the respective participants. We observed a statistically significant difference between the them in favor of the experimental group. We concluded that the results obtained showed that the development of CT in the basic education cycle, without using computational artifacts, can have a positive effect.

In Section II we describe the related works and in Section III, the design of our research including the details of the methodology followed in the study conducted. The results obtained are presented in Section IV with their respective analysis based on statistics. Finally, the conclusions and future works appears in Section V.

II. Background and Related works

It is necessary to help students on realizing that mathematics involves not only finding the right answer to a problem, but also the understanding of the problem and that there is no single solution to it. In this sense, the methodology of problem solving aims to enable students to develop this way of understanding mathematics and its importance for life [14], [15].

Problem solving is not simply to apply purely conceptual math, but rather to learn a true value of mathematics in the people daily lives. Broadening our potential for interpretation, analysis, and organization, before even applying mathematics and the content learned through it [16].

In this context CT appears as an approach to problem solving. Its main goal is to stimulate computing skills that will help students of all ages to stimulate other skills that are needed to improve their ability to solve problems. Wing [1] describes the CT as something built on the strength of the limits of computer science as a science.

The CT is a set of problem-solving skills aimed at the potentialities of the machine, not totally dependent on them, but using it as a working tool to put into practice their way of thinking and formulating solutions to problems. CT can be seen as a way to understand and compute a solution, enabling significant advances in various problem situations. Examples of this applicability are advances in biology, chemistry, and physics.

Wing [2] characterizes the essence of the CT as abstraction, not necessarily clean, elegant and easily algebraic. In the abstraction by the CT every proposal for solving a problem is supported by a set of smaller solutions that are completed. All this formalization is organized in layers of abstraction, and computation automates these layers of abstraction, making feasible the proposed solutions. Therefore, CT exists without the tooling and instrumental computation, because, as said before, it is a human form of thought organization.

Taking into account the application of CT in mathematics, Barcelos et al. [17] present a systematic mapping where papers of the literature focusing on the application of the computation discipline are evaluated. The results presented by the authors indicated that the occurrences of practical applications are more focused on the computational use of computing and the application of the essential concepts for stimulating the problem solving ability have few results.

Lewis and Sha [18] present a correlation between the improvement in student performance in the math discipline after applying an introductory programming course with *Scratch*. During the introductory course, forty-seven students were enrolled. The course is designed to teach programming concepts encompassing mathematical concepts. The exercises of questions used during the course followed features of specific mathematics content, where students, through the Scracth language, performed activities that involved, for example, the practical application of functions. The results obtained by the application of the course allowed the authors to identify a correlation in the way students interpreted and solved the mathematical issues, identifying that the difficulties presented in mathematics were reflected in the programming learning.

Mestre et al. [13] investigated the relationship between CT skills [9] and typical math exercises of questions used in the basic education cycle. The authors defined a mapping between CT skills and fundamental mathematics skills and used it to represent how CT could be associated, in practice, to mathematics. According to this approach a math question is aligned to a CT skill when the text of the question has elements that characterize such a skill. Then, they examined two groups of questions: the first group was composed by 161 questions extracted from PISA 2012 (Programme for International Student Assessment), and the second group consisted of 100 traditional math questions prepared by teachers from Brazilian schools (those participating in the PISA 2012 in Paraíba, Brazil). Analyzing the results, the authors concluded that 6 of 9 CT skills were covered in the questions from PISA 2012 (more skills) and only 4 skills in the traditional questions from teachers (less skills). This means that the two groups of questions had different CT profiles and questions from PISA 2012 were more aligned to CT than the traditional questions available.

Considering a more specific example of the results presented in [13], while in the first group (questions from PISA 2012) the abstraction skill was covered in 100% of the questions, in the second group (traditional math questions from schools), it was only 20%. On the other hand, the skill of problem decomposition was covered in 84% of the second group, while in the first group, it was covered only by 39.13%.

In this paper, it was studied the impact of the use of CT in education by following an independent approach from specific computer disciplines, but introducing CT skills in the disciplines of the basic education cycle, in particular mathematics. The impact was measured by a quasi-experiment [11], [12]. One difference from other related works in the literature in this context is the practical perspective adopted, showing in practice an example on how to use CT in mathematics.

III. Quasi-experiment Design

In this paper we conducted a quasi-experiment to investigate the impact of CT on the problem solving ability of students from Brazilian basic education. In this case, an application of CT in mathematics was considered, addressing the following research question (RQ):

- RQ1: Does applying CT to the elaboration and choice of math questions in basic education has influence over the performance of students in solving problems?

To answer RQ1, the following hypotheses (H) were defined:

- H0: The exposure of students to math questions more aligned to skills developed through CT do not influence students' performance in solving problems.
- H1: The exposure of students to math questions more aligned to skills developed through CT influence students' performance in solving problems less than the traditional questions (less aligned to CT).
- H2: The exposure of students to math questions more aligned to skills developed through CT influence students' performance in solving problems more than the traditional questions.

The evaluation metric (response variable) to answer RQ1 was:

- Final Performance (FP): This variable is the final performance of students in a problem solving test applied to all participants during the study.

A. Participants Selection

The participants of the study were selected from one of the Brazilian schools submitted to PISA 2012, an international test that focuses on measuring the problem solving ability of young students. In this case, our sample comprised two distinct classes of the 8th grade, chosen at random one to be the control group (without CT) and the other, the experimental group (with CT). The 8th grade in Brazil is equivalent to the 8th grade of the middle school in USA, involving under fifteen years old students. It is important to emphasize that the methodology of study of the classes involved in the quasi-experiment followed the same model, because it was considered two classes with the same teacher.

The ages of the participants were concentrated in the range of 13 to 14 years old, for both groups, except for one student in the control group who was 16 years old. Then, the age distribution of the groups was equivalent. The national suitable age for the 8th grade is 13 years old [19]. In total, 23 students for each group participated in the quasi-experiment (46 students). Details of age distribution can be seem in Figure 1.

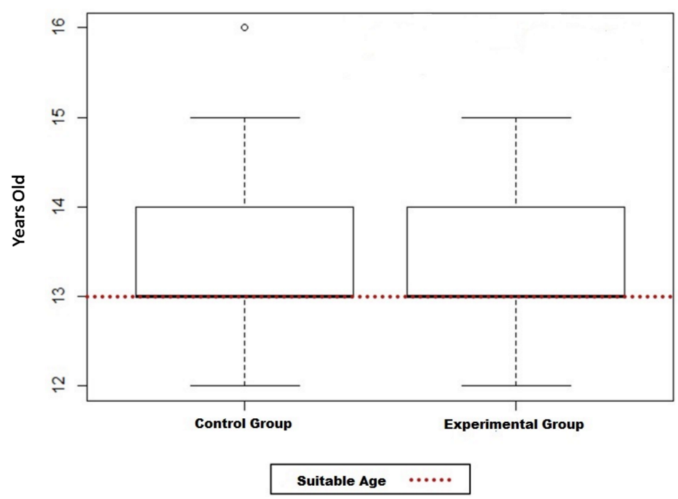

Fig. 1. Age of participants of the study.

Concerning the distribution of students in the two classes involved in the study by gender they are also equivalent, since the two classes present similarity in the number of students of the male and female gender. Details of gender distribution can be seen in Figure 2.

B. Definition of the Math Content

The content used in the quasi-experiment was probability and statistics concepts. This content was selected from a typical math curriculum for the 8th grade in Brazil [20], considering that it had already been studied by the students in the classroom with their math teacher before our study. By taking such a decision it is possible to guarantee that all participants of the study had the same knowledge about the content chosen.

The content definition aims at the selection and elaboration of the questions to be used in this study. It should be emphasized that the math teacher of the classes involved in this study was responsible for the choice of the content.

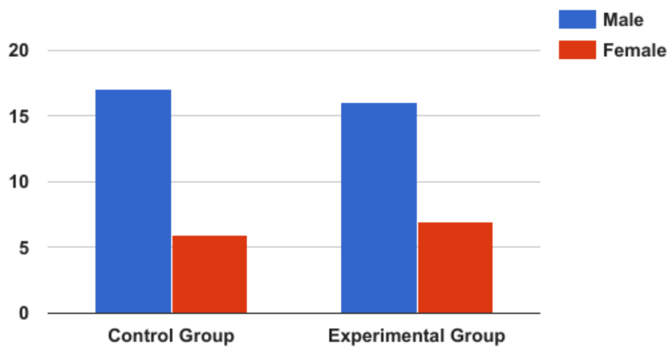

Fig. 2. Gender of participants of the study.

C. Math Questions Sets

It was used two sets of questions: one set with traditional questions prepared by math teachers of the classes involved in the study and, the other set, with questions prepared in order to be more aligned to CT.

Traditional math questions were selected from a number of questions collected from Brazilian schools participating in PISA 2012, particularly 5 schools in Campina Grande, Paraíba. In this case, it was collected 100 questions which are supposed to be used daily by their teachers in math classrooms from 8th and 9th grades. The questions collected covered different content with different complexity levels. A subset of these questions were prepared in order to be more aligned with CT.

It was filtered the traditional questions collected by the target content "probability and statistics notions". After this filtering, the math teacher participating in this study chose 10 questions to be used in the control group (named QS1): 5 questions for classroom and 5 questions for homework. See an example from QS1 in Figure 3.

> The functional staff of a company consists of 45 effective and 25 freelancer. The effective staff 20 are men and 15 freelancer are women. Randomly choosing a person of the company, what is the probability of that person being a woman or freelancer?

Fig. 3. Control group example question.

The questions in QS1 were submitted to a new design; the questions were prepared in order to make them more aligned to the core of CT according to Barr and Stephenson [9]. The 10 questions obtained from this process (named QS2) were used in the experimental group.

In Barr and Stephenson [9] the authors proposed a number of skills that form the core of CT, which are: data collection, data analysis, data representation, decomposition, abstraction, algorithms, automation, parallelization and simulation. Besides, the authors discussed how these skills could be related to the disciplines of basic education.

After understanding the proposal presented in [9], it was proposed a guideline to elaborate math questions based on CT, i.e. questions more aligned with the skills related to CT. This guideline is presented below in 9 steps:

First Step - Definition of Content: Choose the content you want to work according to the national math curriculum guidelines. Another point is the context of the question, i.e. which of day-to-day problems could be worked with that content. This step aims at aligning the content and the problem to be addressed by question in production;

Second Step - Definition of Data Collection: Enable the students to generate or collect the data necessary to solve the question under some guidance and for a particular purpose. Then, the way data will be collection proceed should be defined in the question, for example, using the terms throwing coins and rolling dice;

Third Step - Definition of Data Representation: Represent the data collected for the analysis to be done in the best possible way and the conclusions should be clear. Thus, various forms of data representation can be used for this purpose, for example representing occurrences of coins pitches by histograms;

Fourth Step - Definition of Data Analysis: Define a specif technique in order to analyze the data collected, helping the student on how to analyze what was observed, for example analyzing how often a particular number appears on the scrolling data and in which time the flow of cars increases and decreases;

Fifth Step - Definition of Decomposition Problem: Define a decomposition problem, allowing the student to understand, in practice, mathematical concepts previously learned and which should be taken into consideration in addressing the issue. For example, applying the order of operations in a particular arithmetic expression;

Sixth Step - Definition of Automation Tools: This step involves to stimulate the use of tools to automate the solution of the question. Possible tools in this context are spreadsheets to represent data sets and generate graphs;

Seventh Step - Request to Step by Step: Each student is required to organize the solution of the question as a sequence of steps. It is necessary to advise the student to make clear every step followed to get the solution, not just the final answer;

Eighth Step - Request to Simulation: Specify the issue that the student do simulations with alternate values to solve the problem, for example, simulate different situations of data to observe the final behavior of its solution;

Ninth Step - Joining and Contextualization: The developer should contextualize and join the parts obtained after the execution of the steps described above. The main idea is to have a structured problem that can be divided into several stages where the student must look into it and try to reach a possible solution.

In Figure 4 it is illustrated an example from QS2. Considering the steps identified in the guideline proposed and cited above, the content of this question was the shoes company and the gift voucher (First Step), the students should collect data by the table with the voucher quantity (Second Step), the representation of data by bar chart (Third Step), analyze the data using the probability (Fourth Step), the problem of decomposition are represented by the calculate math (Fifth

Step), automation is defined in the question header using the calculator (Sixth Step), The request of step by step in their header question (Seventh Step), the simulation is performed by means of the calculation request again for an insert of another voucher (Eighth Step). Finally, the joining of all parts in a logical way (Ninth Step).

Lottery

NOTE. Please read the entire question before you start answering. The use of a calculator is allowed, however, the entire process must be registered and logically organized.

Jurandir works in a shoe sales company. Every end of the month, the day of payment to employees, the manager raffle a gift voucher for the employee of the month.

Jurandir was chosen the employee of the month for getting hit the sales target. The bag, the gift voucher are distributed according to the following information.

Voucher	Quantity
50 R$	5
100 R$	3
150 R$	2

First, start representing this information in a bar chart. After this representation, calculate the probability that Jurandir will need to earn 150 R$?

If the manager decided to insert another 150 real gift voucher, the probability of winning 150 Jurandir real increase? Show your results.

Fig. 4. Experimental group example question.

D. Training and Data Collection

After defining the participants and the two sets of questions (QS1 and QS2) used in the quasi-experiment, its execution was divided in three phases: 1) training of the participants in classroom; 2) training of the participants by means of a homework; 3) application of a test based on the PISA 2012 questions to evaluate the participants.

The training in classroom lasted 60 minutes and consisted of making the participants of the control group and experimental group answer an exercise of 5 questions from QS1 and QS2, respectively. Each group worked in a different classroom, in two distinct moments of the same date (09/30/2015), guided by the same math teacher and under our supervision. The role of the teacher was to explain the exercise of questions (QS1 and QS2) and clear the participants' doubts about the questions been worked.

After the training in classroom the participants were oriented to solve more questions given as homework from 09/30/2015 to 10/07/2015 (one week). During this phase the participants could only interact with each other to clear their doubts with no input from the teacher. The homework was delivered by the participants at 10/07/2015, the date defined to start the third phase of the quasi-experiment study.

The third phase started in 10/07/2015, when it was applied a test on problem solving with all participants of the study. This test consisted of 60 minutes of a supervised individual evaluation where participants should solve, without reference material or the teacher assistance, an exercise of 5 questions related with the same content worked in the training phases and randomly selected from the sample provided by PISA 2012. It is important to emphasize that both the control and experimental groups were submitted to the same test.

The performance of the participants in the test was measured as the number of correct answers in the test and the results will be presented in the next Section. The study lasted for 2 weeks, including the execution of the three phases described above, being one hour for the training of the participants in classroom, one week for the training of the participants by means of a homework and one hour for the application of the test based on the PISA 2012 questions.

IV. Results and Discussion

A. Quantitative Analysis of Student's Performance

The first result of interest is the time average spent by each group on solving the test based on PISA 2012, which were 24.56 min and 33.63 min, respectively for the control and experimental group.

The participants in the experimental group spent more time (about 10 minutes longer) on the test than the others in the control group. These results suggest that the time of concentration and attention to solve the questions was greater in the experimental group, resulting in more correct answers by this group.

The evaluation metric considered in our study regards the final performance (FP) obtained by each participant in the test, that is the number of questions successfully solved. An overview of the data collected with the test is shown in Figure 5.

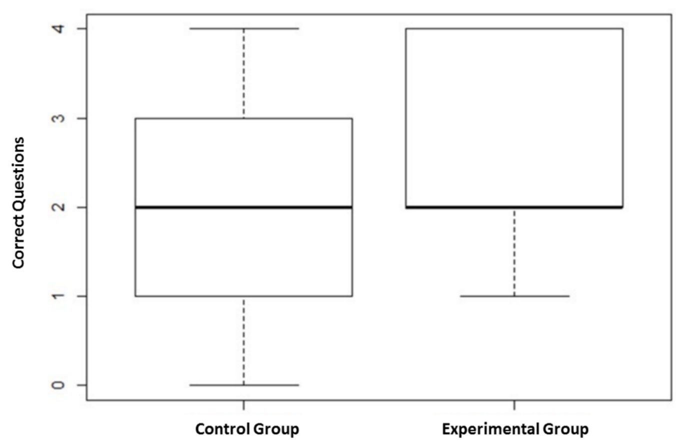

Fig. 5. Student's final performance.

It was observed a greater variation above the median performance in the experimental group. This means that most participants in this group have performed above the average. In the control group, the variations of the performance were equivalent, either up or down the average. Note that, in the experimental group all participants solved at least one question successfully, unlike the control group.

Analyzing the distribution of FP in the two groups, which is represented in ascending order for each group, it was observed that the students in the experimental group outperformed those in the control group (see Figure 6).

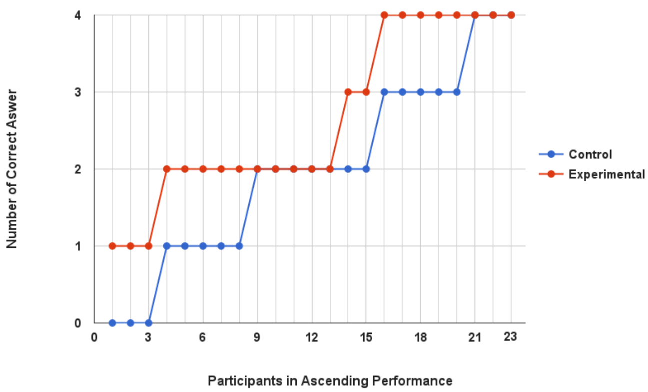

Fig. 6. Student's final performance distribution.

These results show evidence that this improvement was stimulated by the trainings in the classroom and homework. Therefore, the questions more aligned to CT may help on developing the students problem solving ability.

In order to verify whether the differences in the results of the groups were really significant or not, a detailed statistical analysis was carried out. In this sense, the first step was to verify the normality of the data and, according to the results, choosing the appropriate statistical methods. The results of normality tests are shown in Table I.

TABLE I. NORMALITY TEST.

Groups	p-value	*a* (alpha) = 0.05
Control (C)	0.07828	$> a$ (Normal)
Experimental (E)	0.000831	$< a$ (Not normal)
(C U E)	6.115e-05	$< a$ (Not normal)

As the results of normality tests do not tend to normality of the sampling distribution, it was chosen non parametric tests. This choice was made with the purpose of increasing the degree of significance of the results. The results of the non parametric tests on the final performance of both groups are shown in Table II (23 students for each group).

TABLE II. RESULTS FOR THE WILCOXON TEST.

Hypothesis	p-value	$a = 0.05$
FP Experimental = FP Control	0.09135	$> a$ (Rejected)
FP Experimental < FP Control	0.9543	$> a$ (Rejected)
FP Experimental > FP Control	0.04568	$< a$ (Accepted)

According to the results of the statistical tests applied, it is possible to reject the H0 and alternative hypotheses H1. Note that the experimental group achieved a better performance than the control group accepting H2.

It was also analyzed the effect of the method applied to the experimental group on the results obtained. The result of another statistical test showed that the method had a medium effect on the participants performance. Such results are presented in Table III.

TABLE III. RESULTS FOR THE COHEN'S D TEST.

Cohen's Standard	Effect Size	d
Medium	0.5 - 0.7	0.523537

The Cohen's effect proposes a set of the interval number to compare the effect of the method applied. To analyze this effect it was compared the result d with an effect range that characterizes it. The effect intervals are: Small (0.1 - 0.4), Medium (0.5 - 0.7) and Large (0.8 - 1.9).

B. Qualitative Analysis of Student's Performance

The qualitative analysis consisted on the evaluation of the study from the point of view of its participants with the purpose of identifying which features had a greater influence on their performances. These data were collected by means of a questionnaire applied at the end of the quasi-experiment.

The first point analyzed was the impression of the students about the similarity between the questions in the two groups (QS1 and QS2) and the questions daily used in classroom. From Figure 7 it is possible to infer that students who worked with QS1 and QS2 identified some similarity with the questions daily used in classroom. Besides, the group that worked with the new questions (QS2) noticed less similarity with the questions commonly used by the teacher.

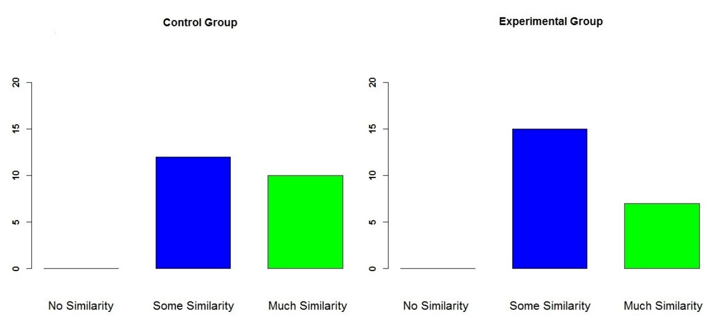

Fig. 7. Similarity with the questions daily used in classroom.

The second point analyzed was the impression of the students about the similarity between the questions in the two groups (QS1 and QS2) and the questions of the final test. In the two groups, the data collected indicated that the questions used in the quasi-experiment were somewhat similar to those in the final test. However, it is necessary to emphasize that participants in the experimental group reported to a greater extent the questions are very similar (see Figure 8).

Another point analyzed was the amount of help required to solve the questions during the training phases. Figure 9 illustrate these results. Observed that the experimental group required more help from colleagues and teacher to solve the questions. In contrast, in the control group most participants did not require any help with the questions. This was probably because the students in the control group worked daily with questions like that. Regarding the assertion of the experimental group on the similarity of QS2 with the questions daily used in classroom, they needed help to solve the questions.

Another interesting point was the impression of the impact of the training in classroom on the final performance of the

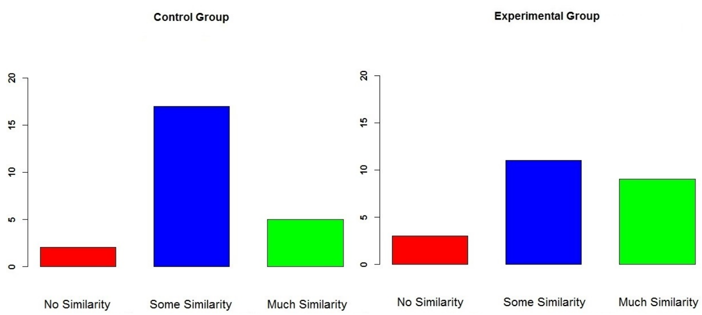

Fig. 8. The final test questions similarity.

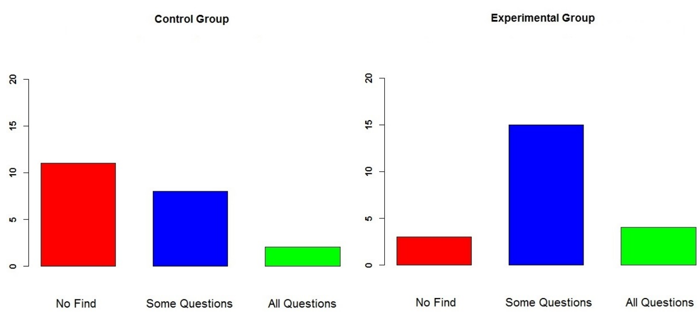

Fig. 9. Amount of help for solving the questions.

participants, i.e. whether the classes on problem solving before the final test helped on their final performance. According to the data collected, the impact was positive for both groups as illustrated in Figure 10. However, it was observed that for the experimental group the impact was even better.

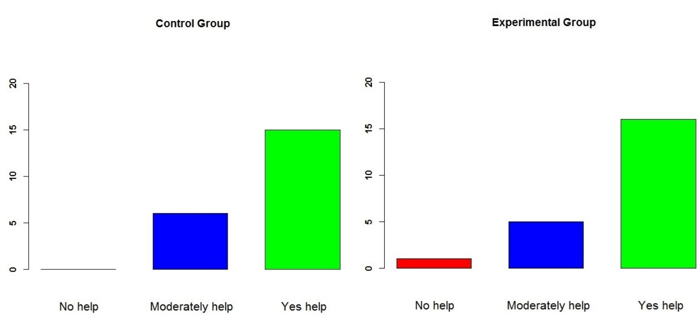

Fig. 10. Classroom training contribution.

Regarding the impact of the training by means of the homework, the impact was again positive (see Figure 11). However, most students in the experimental answered ”Yes help” as opposed to the control group who answered ”Moderately help”. This suggests that there is evidence that the questions used by the experimental group contributed more to the best performance in the final test.

Finally, it was analyzed the impression on the difficulty of the questions in the final test. As illustrated in Figure 12, most participants in the control group classified the questions at an average level of difficulty, while in the experimental group the participants found the questions easy. Such a result confirm those on the impression on the contribution of the training phases, specially for the experimental group that strongly agreed on the positive impact of the training phases and found

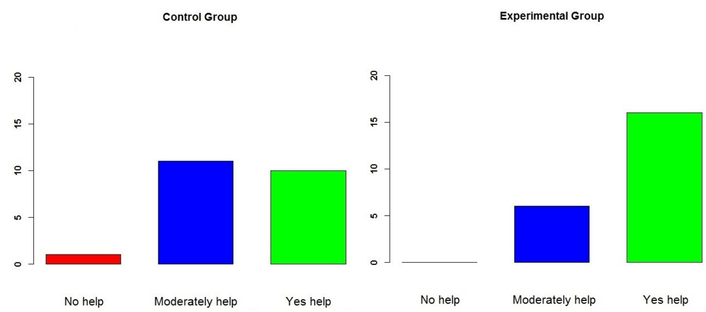

Fig. 11. Homework training contribution.

less difficulty to solve the final test.

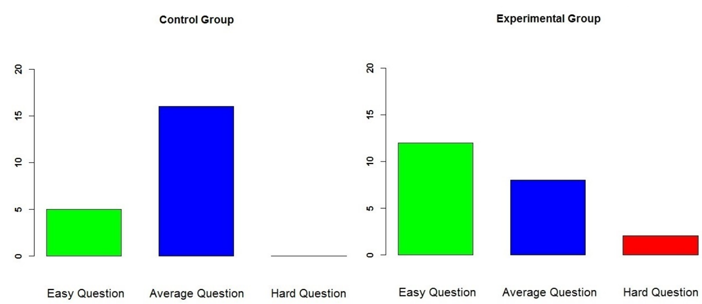

Fig. 12. The final test difficulty.

These results further contribute to the evidence presented above. The training questions in the classroom and homework contributed more to improve the performance of the experimental group compared to the control group in the final test.

The results on the quasi-experiment bring evidence that developing CT in conjunction with mathematics can have an effect, in the long term, on the students performance in terms of problem solving ability. To be more confident on that it is necessary to make new analysis considering, for example, a longitudinal study with different students .

C. Threats to Validity

During the preparation of this research it was identified threats to validity, which were mitigated in accordance with the strategies described below:

- External Validation: despite our efforts to expand the scope of the questions prepared, it is still premature to generalize our framework for preparing math questions. Therefore, it was suggested that new questions are prepared and new evaluation are performed.
- Internal Validation: it was performed a quasi-experiment in two classes of the same level of schooling and applied normality tests to make the statistical results more robust. In addition, to avoid bias in the groups involved in the quasi-experiment it was analyzed how the classes were equivalent in terms of age, gender and methodology of teaching adopted in the classroom, which prove to be true. But it is necessary in future studies an analysis of the previous performance in problem solving of the participating students.

V. Conclusion and Future Works

The results presented in this paper bring statistical evidences that the exposure of students to math questions more aligned with the skills developed through CT influence positively the performance of the students on solving problems. This allowed a positive answer to the research question proposed for this work.

Regarding the quality analysis of the students perception on the quasi-experiment, we observed that the questions of the experimental group influenced the students during the training and their final performance.

The main goal of this work was to contribute on the continuous progression of this area from a practical point of view, providing more evidences on the benefits of developing CT in education as a mechanism to stimulate the students ability on problem solving. A work in progress in this sense concerns performing a content analysis of the drafts produced during the quasi-experiments in the training in classroom and homeworks. The drafts may help on understanding how the students answer the questions proposed to them.

Another future work regards a refinement of this experimental study. The idea is to apply the methodology adopted in the quasi-experiment for a longer period of time in order to strengthen the current results. Students should learn for life, not just to solve repetitive problems.

Another point to be addressed concerns the two approaches pointed out in the literature to develop CT in education: by means of specific disciplines in computing or a joint solution with the disciplines of basic education. Which approach is more effective and for which scenarios?

References

[1] J. M. Wing, "Computational thinking," *Communications of the ACM*, vol. 49, no. 3, pp. 33–35, 2006.

[2] ——, "Computational thinking and thinking about computing," *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 366, no. 1881, pp. 3717–3725, 2008.

[3] T. S. Barcelos and I. Silveira, "Relações entre o pensamento computacional e a matemática através da construção de jogos digitais," *Anais do XII SBGames*, 2013.

[4] M. Resnick, J. Maloney, A. Monroy-Hernández, N. Rusk, E. Eastmond, K. Brennan, A. Millner, E. Rosenbaum, J. Silver, B. Silverman *et al.*, "Scratch: programming for all," *Communications of the ACM*, vol. 52, no. 11, pp. 60–67, 2009.

[5] T. Bell, J. Alexander, I. Freeman, and M. Grimley, "Computer science unplugged: School students doing real computing without computers," *The New Zealand Journal of Applied Computing and Information Technology*, vol. 13, no. 1, pp. 20–29, 2009.

[6] T. Carvalho, D. Andrade, J. Silveira, V. Auler, S. A. da Costa Cavalheiro, M. Aguiar, L. Foss, A. Pernas, and R. Reiser, "Discussing the challenges related to deployment of computational thinking in brazilian basic education." in *WEIT*, 2013, pp. 111–115.

[7] J. T. Jenkins, J. A. Jerkins, and C. L. Stenger, "A plan for immediate immersion of computational thinking into the high school math classroom through a partnership with the alabama math, science, and technology initiative," in *Proceedings of the 50th Annual Southeast Regional Conference*. ACM, 2012, pp. 148–152.

[8] A. Settle, D. S. Goldberg, and V. Barr, "Beyond computer science: computational thinking across disciplines," in *Proceedings of the 18th ACM conference on Innovation and technology in computer science education*. ACM, 2013, pp. 311–312.

[9] V. Barr and C. Stephenson, "Bringing computational thinking to k-12: what is involved and what is the role of the computer science education community?" *ACM Inroads*, vol. 2, no. 1, pp. 48–54, 2011.

[10] A. Lima, W. Andriola, N. de Souza, T. E. da Silva, and Z. L. Viana, "A mixed pedagogical method to improve teaching and learning in brazilian computing area undergraduate courses," in *Frontiers in Education Conference (FIE), 2015. 32614 2015. IEEE*. IEEE, 2015, pp. 1–9.

[11] W. R. Shadish, "Revisiting field experimentation: field notes for the future." *Psychological methods*, vol. 7, no. 1, p. 3, 2002.

[12] W. R. Shadish, T. D. Cook, and D. T. Campbell, *Experimental and quasi-experimental designs for generalized causal inference*. Wadsworth Cengage learning, 2002.

[13] P. Mestre, W. Andrade, D. Guerrero, L. Sampaio, R. da Silva Rodrigues, and E. Costa, "Are typical math problems related to computational thinking?" UFCG, Tech. Rep., 2015.

[14] F. K. Lester Jr *et al.*, "The role of metacognition in mathematical problem solving: A study of two grade seven classes. final report." 1989.

[15] A. H. Schoenfeld, "Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics," *Handbook of research on mathematics teaching and learning*, pp. 334–370, 1992.

[16] C. d. SILVA, "Filho, mgs matemática: resolução de problemas," *Brasília: Liber Livro*, 2011.

[17] T. Barcelos, R. Muñoz, R. V. Acevedo, and I. F. Silveira, "Relaçoes entre o pensamento computacional e a matemática: uma revisao sistemática da literatura," in *Anais dos Workshops do Congresso Brasileiro de Informática na Educação*, vol. 4, no. 1, 2015, p. 1369.

[18] C. M. Lewis and N. Shah, "Building upon and enriching grade four mathematics standards with programming curriculum," in *Proceedings of the 43rd ACM technical symposium on Computer Science Education*. ACM, 2012, pp. 57–62.

[19] N. Farenzena, "A emenda da obrigatoriedade: mudanças e permanencias," *Retratos da Escola*, vol. 4, no. 7, pp. 0–0, 2012.

[20] M. I. LARANJEIRA, "Parâmetros curriculares nacionais: Matemática," *Brasília: MEC/SEF*, p. 37, 1997.